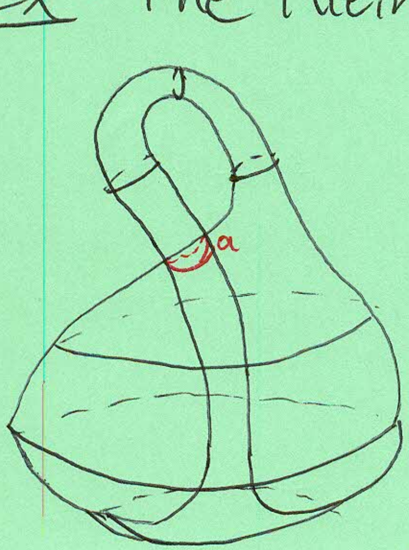
Def: A surface with boundary is <u>orientable</u> if it has 2 distinct sides. A closed surface is <u>orientable</u> if it has a distinct inside and outside.

Ex:

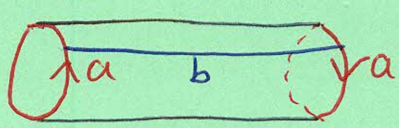
	Orientable	Non-orientable
Surface w/ boundary	Cylinder	Möbius Band
Closed Surface	Torus	Klein Bottle.

We'd now like to represent non-orientable as polygons as well.

Ex: The Klein Bottle $\mathbb{K}$

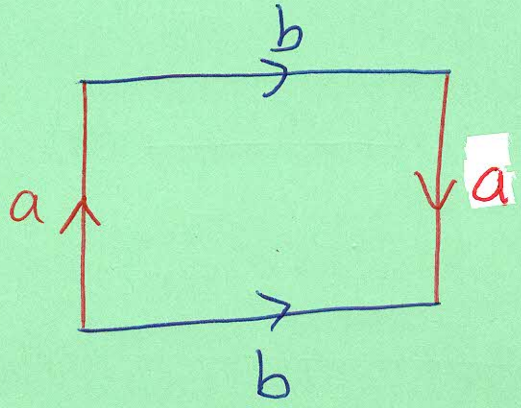


If we cut along what appears to be the self intersection, we get a cylinder like



Notice the direction on one end switches because we have to pull from the inside of the bottle.

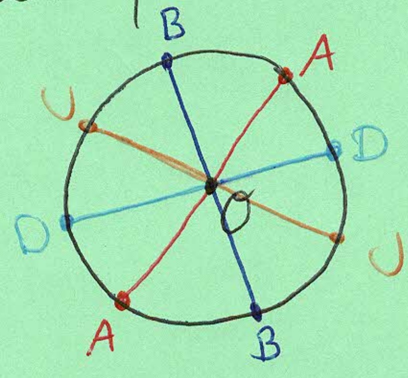
Cutting along the blue line b, we get a square



The final basic example of a non-orientable closed surface is the real projective plane, which we will call $\mathbb{P}$ ($\mathbb{RP}^2$ is better though).

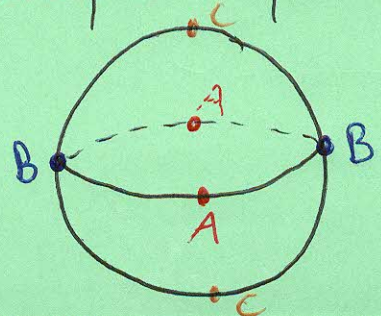
The two easiest ways to think of this are

1) Identify antipodal points on the boundary of a disk



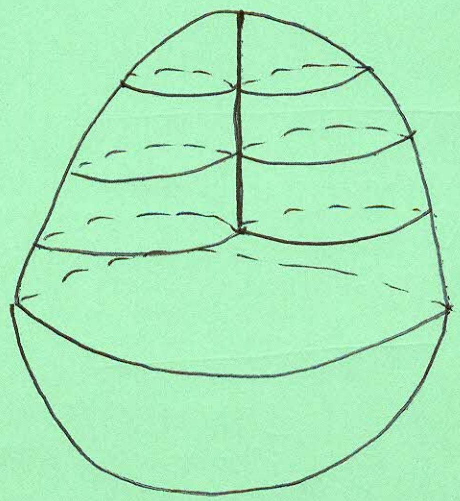
etc.

2) Identify antipodal points of a sphere



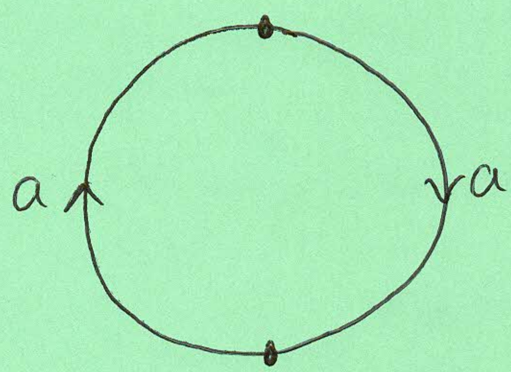
etc.

One way to visualize this surface is



(This visualization is called the <u>cross cap</u>.)

Using the idea that we're identifying opposite points on the boundary of a disk, we can see it as



On the picture above, the dots match up and correspond to the bottom of the vertical line, and said line is what the edge a becomes.

The Euler Characteristic

Now that we have a way to think of a surface as a polygon with edge identifications, we have a simple way to assign a number to a surface.

Let's begin by defining some terms:

A <u>vertex</u> it a specified point (0-dim'l)

An <u>edge</u> is a specified line or curve (1-dim'l)

A <u>face</u> is a shape bounded by edges (2-dim'l) (usually a polygon or a circle).

Let $V = \#$ vertices

$E = \#$ edges

$F = \#$ faces

<u>Def</u>: Let Σ be a surface with a pattern of vertices, edges, and faces drawn on it. The <u>Euler characteristic</u> is $\chi(\Sigma) = V - E + F$